Construction of C^2 Cubic Splines on Refined Triangulations

Carla Manni

Splines on triangulations have widespread applications in many areas, ranging from finite element analysis and physics/engineering applications to computer graphics and entertainment industry. Piecewise polynomials of degree 3 of class C^2 are very appealing because they couple the low degree with a smoothness which allows to efficiently address several problems.

When dealing with a general triangulation, to obtain C^2 smoothness in a stable manner, one must use polynomials of degree 9 on each triangle. An alternative is to use lower-degree macro-elements that subdivide each triangle into a number of subtriangles (or more generally subdomains). The most common macro-structures are the Powell-Sabin split and the Clough-Tocher split. The minimum degree to get C^2 smoothness is 5 on the Powell-Sabin split, while piecewise polynomials of degree 6 are needed to achieve C^2 smoothness on the Clough-Tocher split.

In this talk we consider a C^2 cubic spline space that can be defined on any given triangulation suitably refined and we construct a simplex-spline basis for it. This space ensures full approximation power and any element of the space admits a local construction. Besides computational efficiency, the provided simplex-spline basis possesses all the most important properties we wish for when dealing both for geometric modelling and approximation.

The talk is based on a joint work with Tom Lyche and Hendrik Speleers.